Lecture 4

Fluid Statics

1 Pressure distribution in the Fluid

2 Hydrostatic pressure

3 Pressure Measurement

Fluid statics is that branch of mechanics of fluids that deals primarily with fluids at rest. Since individual elements of fluid do not move relative to one another, shear forces are not involved and all forces due to the pressure of the fluid are normal to the surfaces on which they act. With no relative movement between the elements, the viscosity of the fluid is of no concern. At first it will be examined the variation of pressure throughout an expanse of fluid.Then it will be studied the forces caused by pressure on solid surfaces in contact with the fluid, and also the effects (such as buoyancy) of these forces in certain circumstances.

1 Distribution of Pressure in the Fluid

Consider a small cylinder of fluid PQ as illustrated in Fig.(1) [2]. If the fluid is at rest, the cylinder must be in equilibrium and the only forces acting on it are those on its various faces (due to pressure), and gravity. The cross sectional area δA is very small and the variation of pressure over it therefore negligible. Let the pressure at the end P be p and that at the end Q be $p + \delta p$. The force on the end P is therefore $p\delta A$ and the force on the end Q is $(p + \delta p)\delta A$. If the length of the cylinder is δl its volume is $\delta A\delta l$ and its weight $g\rho\delta A\delta l$ where ρ represents the density and g the acceleration due to gravity. Since no shear forces are involved in a fluid at rest, the only forces acting on the sides of the cylinder are perpendicular to them and therefore have no component along the axis. For equilibrium, the algebraic sum of the forces in any direction must be zero. Resolving in the direction QP:

$$(p + \delta p)\delta A + g\rho\delta A\delta l \cos\Theta = 0 \tag{1}$$

Now if P is at a height z above some suitable horizontal datum plane and Q is at height $z + \delta z$, then the vertical difference in level between the ends of the cylinder is δz

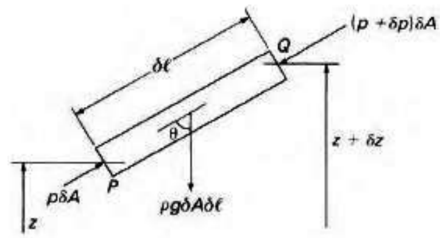

Figure 1:

and δl cos *Theta* = δz. Equation (1) therefore simplifies to

$$\delta p + g\rho\delta z = 0 \qquad (2)$$

and in the limit as $\delta z \to 0$

$$\frac{\partial p}{\partial z} = -g\rho \qquad (3)$$

The minus sign indicates that the pressure decreases in the direction in which z in- creases, that is, upwards. If P and Q are in the same horizontal plane, then $\delta z = 0$, and consequently δp is also zero whatever the value of ρ. Then, in any fluid in equilibrium, *the pressure is the same at any two points in the same horizontal plane*, provided that they can be connected by a line in that plane and wholly in the fluid. In other words, a surface of equal pressure (an isobar) is a horizontal plane.
A further deduction is possible from (3). If everywhere in a certain horizontal plane the pressure is p, then in another horizontal plane, also in the fluid and at a distance δz above, the pressure will be $p + \frac{\partial p}{\partial z}\delta z$. Since this pressure also must be constant throughout a hor- izontal plane, it follows that there is no horizontal variation in $\frac{\partial p}{\partial z}$, and so, by (3), neither does ρg vary horizontally. For a homogeneous incompressible fluid this is an obvious truth because the density is the same everywhere and g does not vary horizontally. But the result tells us that a condition for equilibrium of any fluid is that *the density as well as the pressure must be constant over any horizontal plane*. This is a foundation for the law of connected (communicating) vessels.

The pressure on the horizontal plane that goes through the to vessels which are con- nected, that it is means that to different points one can to join by the curve that always stay with the same fluid, is the same.

This is why immiscible fluids of different densities have a horizontal interface when they are in equilibrium.
There are, then, three conditions for equilibrium of any fluid:

1. The pressure must be the same over any horizontal plane;

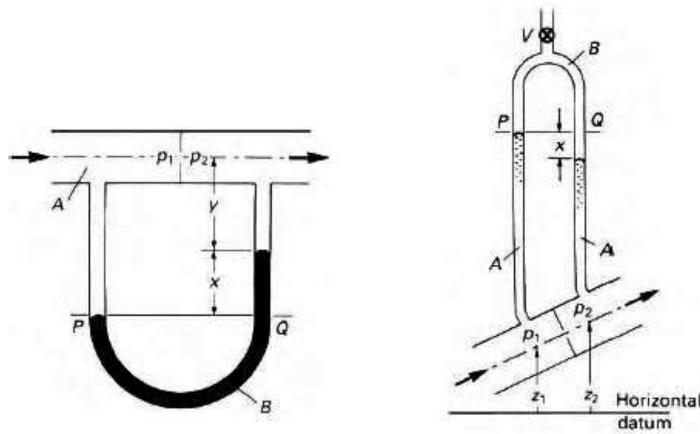

Figure 2: The pressures by virtu of the connected vessels law, at the points P and Q are the same. On the left U-tube manometer, on the right inverted U-tube manometer.

2. the density must be the same over any horizontal plane;
3. The pressure varies only in the vertical z direction as $dp/dz = -g\rho$

2 Hydrostatic pressure

Integration of eqn (3) for a homogeneous fluid of constant density gives

$$\boxed{p + g\rho z = const.} \tag{4}$$

The value of the constant in eqn. (4) is determined by the value of p at a point where z is specified. If the fluid is a liquid with a horizontal free surface at which the pressure is atmospheric (p_a) this free surface may be taken as the datum level $z = 0$. For a point at a depth h below the surface, $h = -z$ (since h is measured downwards whereas z is measured upwards) and, from eqn. (4)

$$\boxed{p = p_a + \rho gh} \tag{5}$$

From the formula (5) it is clear that the pressure increases linearly with the depth, whatever the shape of any solid boundaries may be. Equation (5) shows that the pressure at a point in a liquid in equilibrium is due partly to the weight of the liquid. Thus atmospheric pressure is usually effective, even if indirectly, on all surfaces, and over the differences of height normally encountered it is sensibly constant. Consequently it is often simpler to regard atmospheric pressure as the zero of the pressure scale. As we have already mentioned A pressure expressed relative to atmospheric pressure is known as a

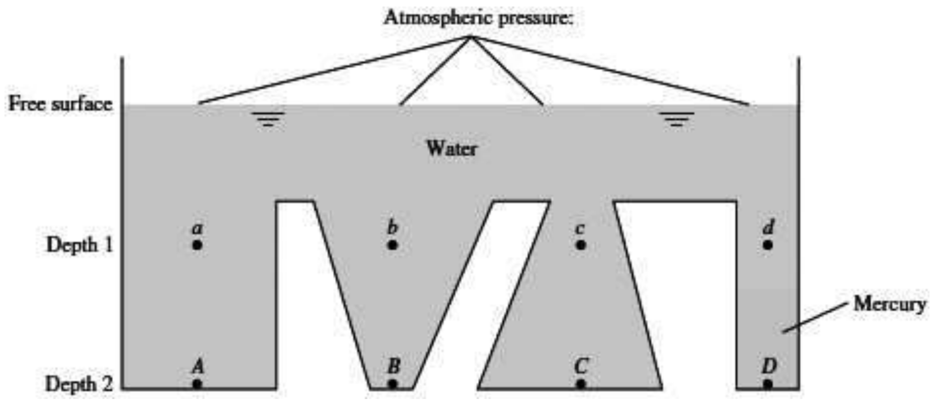

Figure 3: Hydrostatic-pressure distribution. Points a, b, c, and d are at equal depths in water and therefore have identical pressures. Points A, B, and C are also at equal depths in water and have identical pressures higher than a, b, c, and d. Point D has a different pressure from A, B, and C because it is not connected to them by a water path. [1]

gauge pressure. The pressure expressed by equation

$$p = \rho g h \tag{6}$$

is named as the *hydrostatic pressure*.

The direct proportionality between hydrostatic pressure and h for a fluid of constant density enables the pressure to be simply visualized in terms of the vertical distance $h = p/\rho g$. The quotient $p/\rho g$ is termed the *pressure head* corresponding to p. For a liquid without a free surface, as for example in a closed pipe, $p/\rho g$ corresponds to the height above the pipe to which a free surface would rise if a small vertical tube of sufficient length and open to atmosphere known as a *piezometer tube* were connected to the pipe

3 The Measurement of the Pressure

For this lecture I used the following books:

References

[1] F. M. White, 1999. *Fluid Mechanics*, McGraw-Hill.

[2] B. Massey and J. Ward-Smith, 2005. *Mechancis of fluids*, Taylor and Francis.

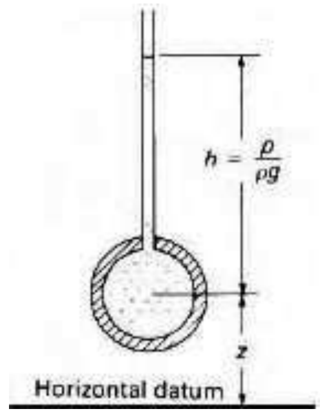

Figure 4: The piezometr. The gauge pressure is visualized in terms of the vertical distance
$h = p/\rho g$.

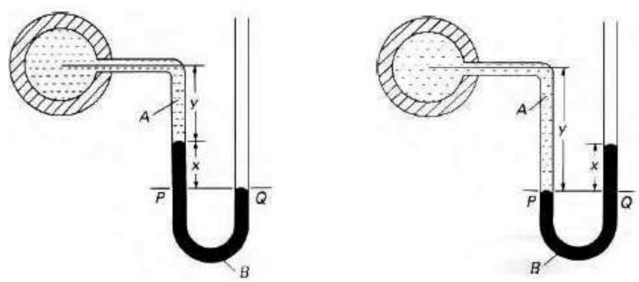

Figure 5: U-tube manometers.

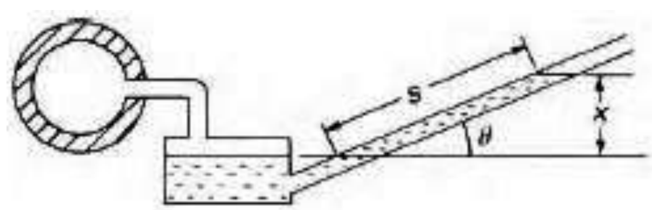

Figure 6: well(vessel)-manometer.